MODELING is the name of the game

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A key to the astounding success of science in discovering the inner workings of natural phenomena has been the development of a powerful way of thinking called *modeling.* To describe and understand the *structure of things*, from raindrops to animals, and the *regularities in natural processes*, from evaporation to locomotion, scientists create *conceptual models* of things and processes. Conceptual models differ from familiar *concrete models*, like dolls and planetariums, in being built out of concepts (or ideas) instead of physical materials. Mathematics supplies the "conceptual tools and materials" for creating models of great clarity, coherence and flexibility. It is through modeling that mathematics can be applied to discover and create order in the world of concrete things. The designs of engineers are conceptual models constructed with "conceptual tools and materials" created by scientists and mathematicians. Thus, modeling is a key to creating new technology. And there is more! Though modeling has achieved the most spectacular results in the natural sciences and engineering, it is equally applicable to social systems of every kind. Indeed, the experience of science tells us that modeling is essential to understand and address the complex problems of modern society. Unfortunately, modeling savvy is in short supply among the leaders in business and government, not to mention the general populace. This can be attributed to serious deficiencies in our educational system.

Since modeling is "the name of the game" in science and technology, it should be the central theme of science education. To appreciate its educational implications, we need a deeper understanding of modeling and its role in the creation and application of scientific knowledge. This article is aimed at that target.

1. What makes knowledge scientific?

Scientific knowledge is distinguished from ordinary knowledge by its *objectivity, precision and structure.*

These distinctions are erratically maintained and sometimes missing altogether in introductory science textbooks and programs. To help students develop thinking patterns that are objective, precise and systematic, we must have a clear idea of how this is achieved in science.

Science is concerned with investigating and describing the properties of natural things and processes. Superficially, a scientific description is similar to an ordinary description, because both employ the subject-predicate form of natural language. But the predicates are fundamentally different: The difference is semantic, arising from a difference in the way properties are attributed to objects. Ordinary predicates tacitly assume comparisons with subjective standards. Thus, when I say, "This book is heavy," I express a comparison of the book with my mental prototype of heavy objects. In contrast, "scientific predicates" express comparisons of things with one another, rather than with subjective prototypes. *Science achieves objectivity by adopting well-defined procedures for systematically comparing objects* with respect to properties. This is the nub of the concept of *measurement* in science. Science courses should help students develop this habit of objective comparison from the beginning and become explicitly aware of the process as soon as they are sufficiently mature. Students cannot be expected to comprehend the structure of science until they have learned to think objectively, in the sense that they can readily distinguish between "objective" properties of physical objects and their own subjective perceptions of them. Undoubtedly many students stumble and fall on this first step to scientific knowledge. Teachers who miss the step cannot be expected to help students over it.

To understand science it is essential to understand the crucial role of objectivity. The objective mode of thinking does more than enable us to make sharp distinctions between inherent properties of concrete things and the attributes of those things in our subjective sensations and states of mind. It enables us to refine and extend our abilities to *discriminate* among observations, to *communicate* information and to *replicate* events, in short to improve the *precision* of our thoughts and actions. Students can learn to appreciate these virtues from "information games" that require them to communicate with greater precision than they are accustomed -- precision sufficient to accurately replicate events.

Objectivity provides the basis for the next step to scientific knowledge: an increase in *precision* by *quantifying* properties, a systematic procedure for representing properties by numbers. Quantification provides the basis for a mathematical description of objects and processes, so it is an essential step in the development of mature science. Quantification is a complex process, and textbooks overlook many of the concepts involved, so it is no wonder that students are invariably confused by it. Consider the *quantification of length*, for example. This involves the construction of a rule (or function) which assigns numbers *a, b, c,* . . . to objects A, B, C, . . ., as expressed by an equation relating objects to numbers:

$$L(A) = a.$$

By juxtaposing objects A and B (physical addition, denoted by $\dot{+}$) we produce ($\doteq$) a new object C, as expressed by the equation

$$A \dot{+} B \doteq C.$$

This corresponds to the equation for addition of numerical values:

$$a + b = c,$$

provided physical addition is analogous to numerical addition, as expressed by the equation

$$L(A \dot{+} B) = L(A) + L(B).$$

This correspondence between "physical addition" and numerical addition is crucial to quantification and measurement not only of length but of any other property, though it is entirely tacit in the science textbooks at every grade level. Moreover, a complete explication of "quantification" requires precise operational definitions of L, $\dot{+}$ and $\doteq$.

Students frequently confuse physical equivalence $\doteq$ with numerical equivalence =, or physical additivity $\dot{+}$ with numerical additivity +. To detect such mistakes it would be desirable to design tests for every component of the operational definition and quantification of primary variables in science. Furthermore, activities should be designed to make sure that every aspect of quantification is actually taught when it is needed. It would not be appropriate to present students with a formal theory of quantification, but formal theory is necessary to ascertain precisely what ought to be taught.

The development of scientific knowledge goes hand-in-hand with the development of suitable language and other symbolic devices to assist scientific thinking and record accumulated knowledge. Natural languages like English are already powerful tools for thinking. But for the purposes of science the natural language must be *refined* to meet scientific standards of objectivity and precision as well as *extended* to express new scientific concepts. Mathematical symbols, in particular, have been created to express concepts of order and structure. Indeed, mathematics has been aptly described as "the science of patterns."

Natural language is often ambiguous about distinctions between concepts and things, leading to muddled thinking and even superstition among the unwary. To maintain the objectivity of science, it is essential to make a sharp distinction between the *conceptual world* of ideas (or concepts) and the *concrete world* of things. Mathematical concepts (such as number, set and function) reside solely in the conceptual world, though they are often used to create conceptual models of things in the concrete world. Thus, the *numeral* '8' (a thing) designates the *number* "8" (a concept) which, among other things, represents the count of planets in a model of the solar system.

2. Conceptual Models

Scientists go beyond mere description by developing *validated conceptual models* of natural things. Such models are objects in the conceptual world, which *represent* things and their properties in the concrete world. Modeling begins with *description*: creating a list of descriptive variables (or *descriptors*), each representing a *property* of the thing (or class of things) in question. Descriptors are concepts while the properties they represent are inherent in the concrete world. The natural language is commonly used to describe things by listing their properties, but the crucial distinction between the concrete properties and the (conceptual) descriptors that represent them is seldom made; moreover, the scientific standards of objectivity and precision are not met. When these scientific standards have been satisfied and the fidelity of the description has been empirically established, a conceptual model is said to be *validated.*

Like the property "length," which can be attributed to solid objects, the representation of many properties can be *quantified* by adopting appropriate rules for comparing things that possess them. Then the corresponding descriptors can be assigned numerical values along with a *unit* (like foot or meter) representing the standard for comparison. All the descriptors of physical properties (such as mass, position, temperature and force) have been quantified. The advantages of quantification are twofold: First, it is a means for describing subtle variations in properties. Second, it enables the representation of "*natural laws*" relating different properties of things as *invariant* mathematical equations relating descriptors. Here "invariant" means independent of the reference standards chosen for quantification.

Scientists have discovered that the descriptors of natural things fall into three broad classes: physical, chemical and biological, each with its own system of natural laws. Without delving into specifics of the sciences, we note some general features of properties and laws that have been established in all the sciences and so enable us to refine our general concept of conceptual model.

The properties of things are of two general types: *intrinsic* and *interactive.* Intrinsic properties belong to the thing by itself, while interactive properties are shared with other things. Some intrinsic properties (such as the physical property of mass or the chemical property of valence) may have fixed values in a model while other properties change. It is convenient to express this distinction in the descriptors by introducing the term *object variable* for a fixed property and *state variable* for a changeable property. Thus, an object variable has a constant value for a particular thing, whereas it may take different values for different things. It is often called a *parameter* of the model, as it can be adjusted so the same model describes a whole class of different things. *Behavior* of a thing is represented by changes of its state variables.

The descriptors of interactive properties are called interaction variables or just *interactions*. A thing that *acts on* another thing is called the *agent* of the action. Two things that act on one another are said to *interact*. Thus, *interactions* (Also called *connections, links, bonds,* or *couplings*) are mutual (or shared) properties of things. Interactions influence (change or constrain) the object variables of a thing according to natural laws. Indeed, it is usually by observing or experimenting with changes in object variables that interactions are discovered and characterized by scientists. Scientists have identified and modeled a great variety of interactions, including physical *forces*, chemical and social *bonds*, *flows* of energy and information. The set of things with which an individual thing interacts is called the *environment* of the thing. A crucial step in understanding the behavior of a thing is identifying the agents in the environment that interact with it. Failure to realize this is a common source of student confusion in slipshod science courses.

Natural laws are of two general types, best described in terms of their conceptual representations: (1) *Laws of change*, which specify how state variables change; (2) *Interaction laws*, which specify relations between state variables and interactions. When the descriptors are quantitative, as in physics, differential calculus ("the mathematics of change") can be employed to give a precise formulation of the laws of change, and this can be combined with the interaction laws to give *differential equations of change* for the model thing. Then the conceptual model can be called a *mathematical model*.

To summarize, a *conceptual model* in science is defined by specifying the following:

(1) *Constituents*:

Names for the thing of interest and the things in its environment.

(2) *Descriptors*:

Object variables,
State variables,
Interactions.

(3) *Laws*:

Laws of change,
Interaction laws.

(4) *Interpretation*:

Relates descriptors of the model to properties of the object.

A great variety of models can be constructed for any given thing, depending on the purposes of the modeler. Scientific theories supply advice on what variables and laws to use. No single model characterizes a concrete thing completely. Nor would such a model be desirable, because its complexity would make it too cumbersome to be useful. One of the most important objectives of modeling is to *focus* on the most significant or relevant properties of a thing by constructing simple models that eliminate or suppress minor details.

3. Objects and Models with structure

The structure of scientific knowledge reflects the structure of things in the concrete world. Structure is one of the most significant general properties of things, so it deserves special attention. Structure is an abstraction, however, which does not exist apart from some object. To take this into account, it is convenient to introduce the concept of system. *A system is an object with structure*. This means that a system is a complex object composed of other objects referred to as its *constituents* or *parts*. The structure of a system gives it a certain *integrity* or *wholeness*, so it is not just an arbitrary aggregate of objects. The structure is separable into an *internal structure* relating the constituents to one another and an *external structure* relating the constituents to objects in the system's environment. A system is said to be *closed* if it has no external interactions; otherwise it is *open*.

Since there are two distinct kinds of object: conceptual and concrete, there are two distinct kinds of system. *Conceptual systems* inhabit the conceptual world, while *concrete systems* inhabit the concrete world.

Unlike things, concepts do not interact. Though interactions are represented by relations in conceptual models, an interaction is more than a mere relation. Consequently, though a conceptual system can be regarded as a *set* of interrelated conceptual objects, a concrete system is not the same thing as the set of its parts. Sets are concepts, never things, and the assembly of a thing from its parts is not to be confused with composing a set. Being concepts, relations among the constituents are not inherent properties of a conceptual system; they are simply assigned. The assignments are not entirely arbitrary, however, as they must satisfy certain *systemicity criteria*: consistency and coherence, so the whole system has integrity.

Mathematical systems are conceptual systems. Two of the most important are: The *real number system* and *Euclidean geometry*. They contain an infinite variety of objects as subsystems. For example, the integers and rational numbers in the first case; triangles and other geometric figures in the second case. These systems are of immense utility for modeling structures in science and technology as well as everyday life.

We are now prepared to generalize our concept of model to handle structure. We say that system A is a *structural model* of system B if the structure of A is similar (in some respect) to the structure of system B. As before, A is a conceptual model of B if it is a concept. But now we allow other possibilities: We say that A is a *concrete model* of B if A is a thing, and we also allow B to be either concept or thing. When both A and B are mathematical systems, the similarity relation is called an *isomorphism*.

Many kinds of concrete model are used in science. Chemists construct ball-and-stick models of molecules to accurately portray and visualize spatial structure. Biologists use "animal models" of human response to drugs, because they can't experiment on humans, and they don't yet know enough to rely on their tentative conceptual models. *Computer models*, which are embedded in computer programs, are increasingly popular because of the great power and flexibility of computers. The most common kind of concrete model, however, is constructed from a symbol system, such as English, and embedded in "*hard copy*" on paper. This kind of model requires a reader to supply its structure by interpreting the text. Computer models are more powerful not just because they can handle great complexity and perform rapid calculations, but also because the structure of the models can be built right into them, not merely symbolized. Computers are increasingly able to perform "intelligent acts" which could only be performed by the human brain in the past.

Conceptual models are more fundamental than concrete models because they are abstractions, so when the term "model" is used in science, it should be understood as "conceptual model" unless it is designated as concrete. *A concrete model can be regarded as a (conceptual) model embedded in a concrete medium.* It is called a *realization* of the model. Being an abstraction, a conceptual model does not actually exist apart from some realization. The most significant realization is as a *mental model* in the brain of some human. However, computer models in the "brains" of robots are fast becoming rivals. Realizations can take many forms called *representations*. Besides symbolic representations there is a variety of graphical, graphic and pictorial representations. A representation is sometimes identified (or confused) with the model itself. Thus, people speak of "graphical models" or "pictorial models." Instead, one should speak of "graphical representations" or "pictorial representations." A single representation is usually insufficient to express the full content and structure of a scientific model. A family of coordinated representations is required, each particular representation giving one *view* of the model.

We have already noted the essential role of *symbol systems* in the development of mathematics and science. Computers provide a medium for creating new and more powerful representations. One such representation, the computer *simulation*, is the most prominent scientific application of computers today. Simulations have a *temporal* feature that cannot be duplicated in a *permanent* medium like a book. Simulations reveal implications (properties) of complex mathematical models that cannot be discovered in any other way. But a simulation should not be identified (or confused) with the model it represents. As a representation, it gives only one view of the model. Though informative, this view is limited.

This completes our explication of the concept "model" in science. To exhibit the power of this concept, let us review some amazing conclusions about the structure of nature which modeling has produced.

4. Structure of the natural world

We can distinguish two kinds of concrete system: *natural systems*, such as planets, animals, cells and atoms; and *artificial systems* (created by humans), such as watches, books, factories and governments. Unlike a conceptual system, the structure of a natural system is inherent in the system itself. Scientists aim to discover the structures of natural systems and represent them accurately with models. They have accumulated an enormous body of evidence in support of the following claims, which are suitable tenets for a general theory of nature:

(1) *Systemic and spatial structure*. The structure of any concrete system can be separated into systemic and spatial parts, which are connected by natural laws. Interactions of the constituents determine a *systemic structure* for the system. Spatial relations among the constituents determine a *spatial structure* (or *configuration*). The system of natural laws governing spatial structure determines the geometry of space and time. It has been called the *Zeroth Law*, because it applies to every concrete thing and precedes all other laws. The Zeroth Law has been thoroughly investigated and precisely formulated by physicists. In applications outside physics it is often ignored, though, where systemic structure commands greater interest.

(2) *A world of systems.* The natural world is composed of semi-permanent systems which maintain their integrity in suitable environments. Examples of natural systems: atoms, molecules, cells, organisms, populations.

(3) *Level structure.* Natural systems are organized into levels of increasing structural complexity (e.g. atomic level, molecular level, cellular level, . . .). The lowest level consists of a small number of irreducible elementary particles, such as the electron, which are not composed of other things. Thus, elementary particles are the ultimate constituents of all concrete things.

(4) *Resultant and emergent properties*. Every property of a concrete system as a whole is either resultant or emergent. A *resultant property* is a property of the constituents that is inherited by the system. The physical properties of energy and electric charge are resultant. An *emergent property* is a property of the whole system that is not possessed by its constituents. It is a structural property of the system, depending on how the system is assembled. The geometrical property of shape is emergent. Elementary particles and atoms do not have shape. Obviously, the shape of a brick house is not inherent in the bricks but emerges from their assembly. Less obvious is the well-established fact that the chemical properties of molecules emerge from their assembly out of electrons and atomic nuclei, which do not have such properties.

(5) *Structural explanation.* All properties which are not inherited are emergent, so they can be explained as consequences of structure. There is much evidence that this applies even to the property of *life*, that is "being a living thing," though the issue is sure to be under scientific investigation for a long time to come. On the other hand, physicists have reduced interaction properties to four fundamental kinds: gravitational, electromagnetic, strong (or nuclear) and weak. To think that the rich diversity of the natural world may arise from such simple constituents is awe-inspiring.

5. Modeling for everyone

We have seen that modeling is a means to discover and create order in the fabulous world of everyone's experience. The benefits of modeling should be obvious to anyone who knows what modeling is, but a list may be worthwhile for emphasis.

(1) *Focus.* Modeling helps identify essential factors and eliminate irrelevant information.
(2) *Organization*. Modeling organizes complex information systematically and so facilitates memory storage, retrieval and communication.
(3) *Empowerment.* Modeling facilitates planning, so it is a means to effective action.
(4) *Access* to the power of scientific and technical knowledge is available only to those who understand its origin and use through modeling.
(5) *Protection* from pseudoscientific fraud and misinformation comes with the ability to recognize the models underlying scientific claims and arguments.
(6) *Enrichment.* A deep appreciation of the wonders of this world revealed by science comes only with an understanding of the models it creates.

6. Modeling in the math-science curriculum

The preceding discussion supports the view that *modeling is the main activity of scientists.* Most of the discussion was devoted to elucidating the concept of "model," to clarify the purpose and product of modeling. Modeling is a complex activity, or rather, a coordinated complex of four different kinds of activity or *modeling modes*:

(1) *Development.* Constructing a model of some concrete system to meet given theoretical or empirical specifications.
(2) *Analysis.* Analyzing the structure or implications of a given model; for example, by studying simulations.
(3) *Validation.* Evaluating the capability of a model to account for given data or describe/explain given concrete properties and events.

(4) *Deployment.* Applying a given model to describe or design concrete systems or to explain or predict events.

Mathematics often plays a big role in all these modeling activities. In fact, mathematics is essential for powerful modeling. However, much more than mathematics is involved. *Without modeling, mathematics is impotent* - - unable to play its role in organizing experience.

This is not the place for a detailed analysis of the four major modes of modeling and the various cognitive skills they require. Only a general idea of what modeling involves is needed here. With the understanding that modeling is central to science, we formulate the following *general objectives for science education*:

(1) Help students learn the coordinated cognitive skill structures required for modeling.
(2) Familiarize students with basic models in each of the sciences.
(3) Give students rich experience in modeling significant systems in the real world.

Science curricula with modeling as the central theme are yet to be developed, but physics is far along (see references). There is a particular need to spell out the role that modeling should play in elementary school, so let us consider the bearing of ideas in the present paper on the problem.

Elementary math-science curricula do aim to teach basic modeling skills. A typical list of such skills includes: *grouping, ordering, counting, measuring, graphing.* These are often called "math skills", but "modeling skills" is more apt. Children develop these skills by manipulating concrete objects. Counting, for example, must be learned with concrete objects, and its purpose is to determine a concrete property, the *cardinality* of a given set of objects. There is a symbiotic relation between mathematics and modeling: not only does mathematics serve as the tool of choice for modeling; *modeling makes math meaningful.* Long experience with both is required for students to develop the ability to separate mathematical abstractions from concrete experience.

From our present perspective, elementary math-science curricula suffer most seriously from a failure to make modeling the central theme as well as failure to identify basic models with many significant applications. Consequently, instruction is often fragmented and haphazard: students practicing counting, computing and measuring without purpose. Models are needed to coordinate modeling activities toward some goal, toward applications that are meaningful to the students.

It should not be difficult to identify and name a small number of *basic model types* that suffice for all the applications of elementary mathematics. To be specific, we identify two: *inventory models* and *maps.* Inventory models have long been around as a "use class" for mathematics, but they have not been given a name. Maps are familiar to everyone, but few recognize "map" as a fundamental model type.

To *take inventory* of a given system of concrete items is to construct an inventory model. The system is modeled as a container (such as a store or stockroom) and items within the container are regarded as constituents of the system. Taking inventory involves *sorting* (or grouping) the constituents into classes (subsystems) and *counting* the number of items in each class to determine its cardinality. A typical *representation for an inventory model* consists of a list of names (or bins) paired with numeral representing cardinality (multiplicity of contents). Arithmetic addition and subtraction are used to model changes in (the state of) the inventory (model), with physical addition $\dot{+}$ understood to mean "placing in the container."

The inventory model is very useful because it is generic; it can be applied to any system whatsoever. Indeed, "taking inventory" or "cataloguing parts" is usually one of the first steps in modeling a new system. The only mathematics required for inventory modeling are elementary set theory and arithmetic. The model includes nothing about criteria for sorting the items; it requires only that the sorting can be done. The classification could be based on natural properties or some arbitrary assignment (such as price) to the item. An important elaboration of the inventory model is arranging like items into subgroups of singles, tens and hundreds for efficient counting and recording by decimal numerals. Replacement of the tally and Roman numeral systems by the decimal system (only about 300 years ago) is one of the first great contributions of mathematics to business. A short historical account of the revolution it wrought would be appropriate in the curriculum to help draw attention to the significance of notational inventions. The inventory model must be the most widely (and tacitly) used model in business and practical affairs. Once students grasp the idea of an *inventory structure*, they are likely to start seeing inventories everywhere.

The inventory model can be extended in many ways to include other structural properties of interest. In middle school the model should be extended to apply to continuous quantities so it becomes possible to make "energy inventories," for example.

The map is a generic model type because it models spatial structure, which the Zeroth Law says is possessed by every concrete system. Students should recognize architectural plans as well as road maps as models of one type. They should learn that to read a map is to know how the model (map) relates to its referent. They should realize that the purpose of measuring distance or length is invariably to make some kind of a map of an object large or small. Eventually they should come to realize that geometry is the scientific theory on which map making is based.

In implementing the modeling theme by developing, analyzing, evaluating and deploying basic models, the emphasis should be on the concepts of *structure* and *change* and the refinement of these concepts with mathematical tools of increasing power.

6. References

The literature on systems and modeling is vast and confused. A beacon of clarity in this fog is the work of physicist-philosopher Mario Bunge, which has illuminated much of the present article. Bunge has written extensively on systems, models and modeling, including [1979] a systematic survey of systems across *all* the sciences, natural and social. However his work is very sophisticated, so it is recommended only to those who have the time for serious study. The article by Hestenes promotes modeling as the central theme of physics instruction.

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